we define

$$S = \frac{dR}{d\theta} R^T(\theta) \qquad (3)$$

then

$$S^T = R(\theta) \frac{dR^T(\theta)}{d\theta} \qquad (4)$$

(3) & (4) in (2) we get

$$S + S^T = 0$$

$$\therefore \ S \in SS(3)$$

The matrix

$$S = \frac{dR(\theta)}{d\theta} R^T(\theta)$$

is a skew symmetric matrix.

Multiplying both sides of (3), on the right by R

$$S R(\theta) = \frac{dR(\theta)}{d\theta}$$

Ex:

$$R = R_{\hat{x}}(\theta) = \begin{bmatrix} 1 & 0 & 0 \\ 0 & C_\theta & -S_\theta \\ 0 & S_\theta & C_\theta \end{bmatrix}$$

$$S = \frac{dR(\theta)}{d\theta} R^T(\theta)$$

$$= \begin{bmatrix} 0 & 0 & 0 \\ 0 & -S_\theta & -C_\theta \\ 0 & C_\theta & -S_\theta \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 \\ 0 & C_\theta & -S_\theta \\ 0 & S_\theta & C_\theta \end{bmatrix}$$

$$= \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & -1 \\ 0 & 1 & 0 \end{bmatrix} = S(\hat{x})$$

$$\hat{x} = [1 \quad 0 \quad 0]^T$$

$$\boxed{\frac{dR_{\hat{k}}(\theta)}{d\theta} = S(\hat{k}) R_{\hat{k}}(\theta)}$$

$\hat{k}$: axis of rotation

another important property.

Angular Velocity and Acceleration

Suppose that a rotation matrix R is time varing

$$R(t) \in SO(3) \quad \text{for all } t.$$

$$\dot{R}(t) = \frac{dR(t)}{dt} = S(t)\,R(t)$$

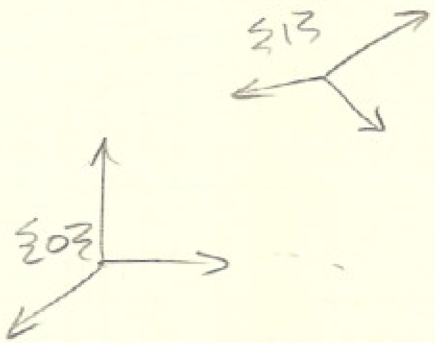
where

$$S(t) = \frac{dR}{dt} R^T(t)$$

$S(t)$ is a skew symmetric matrix, it can be represented by $S(\omega(t))$ for a unique vector $\omega(t)$

This vector $\omega(t)$ is the angular velocities of the rotating frame w.r.t. the fixed frame at time t.

In Summary



$\omega(t)$ is the angular velocity of frame $\{1\}$ w.r.t. $\{0\}$

$\omega(t)$ is such that

$$S(\omega(t)) = \dot{R}(t)\,R^T(t)$$

EX:

$$R(t) = R_{\hat{x}}(\theta(t))$$

$$\dot{R} = \frac{dR}{d\theta} \cdot \frac{d\theta}{dt}$$

$$= \dot{\theta}\, S(\hat{x})\, R$$

$$= S(\dot{\theta}\hat{x})\, R$$

$$= S(\omega(t))\, R$$

$$\boxed{\omega(t) = \dot{\theta}\,\hat{x}}$$